HOSSAM GHANEM

(24) 3.7 Implicit Differentiation(B)

Example 1 60 October 31 , 2011	(4 points) Let $f(x) = x^3 \tan^2 x + \sqrt{\sec(2x)}$. Find $f'(x)$.

Solution

$$f'(x) = 3x^2 \tan^2 x + x^3 \cdot 2 \tan x \sec^2 x + \frac{2 \sec(2x) \tan(2x)}{2\sqrt{\sec(2x)}}$$

Example 2	Find $\frac{dy}{dx}$ if $\frac{xy^3}{1+\sec y} = 1 + y^4$

Solution

$$\frac{xy^3}{1+\sec y} = 1 + y^4$$
$$xy^3 = (1 + y^4)(1 + \sec y)$$
$$xy^3 = 1 + \sec y + y^4 + y^4 \sec y$$
$$y^3 + x \cdot 3y^2 y' = \sec y \tan y \; y' + 4y^3 y' + 4y^3 \sec y \; y' + y^4 \sec y \tan y \; y'$$
$$y'[3xy^2 - \sec y \tan y - 4y^3 - 4y^3 \sec y - y^4 \sec y \tan y] = -y^3$$
$$y' = \frac{-y^3}{3xy^2 - \sec y \tan y - 4y^3 - 4y^3 \sec y - y^4 \sec y \tan y}$$

Example 3	Find $\frac{dy}{dx}$ if $\tan^3(xy^2 + y) = x$

Solution

$$\tan^3(xy^2 + y) = x$$
$$3\tan^2(xy^2 + y) \sec^2(xy^2 + y) \cdot [y^2 + x \cdot 2yy' + y'] = 1$$
$$y^2 + y'(2x + 1) = \frac{1}{3\tan^2(xy^2 + y) \sec^2(xy^2 + y)}$$
$$y'(2x + 1) = \frac{1}{3}\cot^2(xy^2 + y) \cos^2(xy^2 + y) - y^2$$
$$y' = \frac{\frac{1}{3}\cot^2(xy^2 + y) \cos^2(xy^2 + y) - y^2}{(2x + 1)}$$

Example 4

44 December 21, 2008

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0$, where $2y + \sin(xy) = 1$.

Solution

$2y + \sin(xy) = 1$

$at \quad x = 0$

$2y + 0 = 1 \qquad \Rightarrow y = \frac{1}{2}$

$2y + \sin(xy) = 1$

$2\,y\backslash + \cos(xy)\,[y + xy\backslash] = 0$

$2\,y\backslash + y\,\cos(xy) + xy\backslash\cos(xy) = 0$

$y\backslash[2 + x\cos(xy)] = -y\cos(xy)$

$y\backslash = \frac{-y\cos(xy)}{2 + x\cos(xy)}$

at $\left(0\,,\frac{1}{2}\right)$

$y\backslash = \frac{-\frac{1}{2}\cos(0)}{2} = \frac{-\frac{1}{2}}{2} = -\frac{1}{4}$

$y\backslash\backslash = \frac{[2 + x\cos(xy)][-y\backslash\cos(xy) + y\sin(xy)(y + xy\backslash)] + y\cos(xy)\,[\cos(xy) - x\sin(xy)\,(y + xy\backslash)]}{[2 + x\cos(xy)]^2}$

$at \quad \left(0\,,\frac{1}{2}\right) \qquad \Rightarrow\ y\backslash = \frac{-1}{2}$

$y\backslash\backslash = \frac{[2+0]\left[\frac{1}{2}(1)+0\right] - \frac{1}{2}(1)[1-0]}{[2+0]^2} = \frac{1 - \frac{1}{2}}{4} = \frac{\frac{1}{2}}{4} = \frac{1}{8}$

Example 5

43 July 19, 2008

Find an equation of the normal line at $x = 0$ to the graph of

$y^3 + y\sin(x) - \cos(xy^2) = 0$

Solution

$y^3 + y\sin(x) - \cos(xy^2) = 0$

$at \quad x = 0$

$y^3 + 0 - 1 = 0 \qquad \Rightarrow y = 1$

$3y^2y\backslash + y\,\cos x + y\backslash\sin x + \sin(xy^2)\cdot[y^2 + 2xyy\backslash] = 0$

$p(0\,,1)$

$3y\backslash + 1 + 0 + 0 = 0$

$y\backslash = \frac{-1}{3}$

$m = 3 \qquad , \quad p(0\,,1)$

$y - y_1 = m\,(x - x_1)$

$y - 1 = 3x$

$y - 3x - 1 = 0$

Example 6	36 Dec 15, 2005 Find an equation of the normal line at $x = 0$ to the graph of $\sec^2(\pi + x) + \sin(xy) + y = 0$

Solution

$\sec^2(\pi + x) + \sin(xy) + y = 0$
at $x = 0$
$\sec^2 \pi + \sin 0 + y = 0$
$1 + y = 0 \quad \Rightarrow \quad y = -1$
$2\sec(\pi + x) \cdot \sec(\pi + x)\tan(\pi + x) + \cos(xy)\left[y + xy^{\backslash}\right] + y^{\backslash} = 0$
$p(0, -1)$
$2\sec(\pi) \cdot \sec(\pi)\tan(\pi) + \cos(0)\left[-1 + 0\right] + y^{\backslash} = 0$
$0 - 1 + y^{\backslash} = 0$
$y^{\backslash} = 1$
$m = -1 \quad \& \quad p(0, -1)$
$y - y_1 = m(x - x_1)$
$y + 1 = -x$
$x + y + 1 = 0$

Example 7	39 December 14, 2006 Find an equation of the tangent line to the curve of $2y + y^2 \tan x + \sin(x^2 y) - 2 = 0$ at $x = 0$

Solution

$2y + y^2 \tan x + \sin(x^2 y) - 2 = 0$
at $x = 0$
$2y + 0 + 0 - 2 = 0$
$2y = 2 \quad \Rightarrow \quad y = 1$
$2y^{\backslash} + 2yy^{\backslash}\tan x + y^2 \sec^2 x + \cos(x^2 y)\left[2xy + x^2 y^{\backslash}\right] = 0$
$p(0, 1)$
$2y^{\backslash} + 0 + 1 + [0] = 0$
$y^{\backslash} = -\frac{1}{2}$
$m = -\frac{1}{2} \quad \& \quad p(0, 1)$
$y - y_1 = m(x - x_1)$
$y - 1 = -\frac{1}{2}x$
$2y - 2 = -x$
$x + 2y - 2 = 0$

Example 8	52 July 23 , 2011 A [4 Points] Find an equation of the normal line at $x = 0$ to the graph of $xy^2 + y\sin(x) + y^3 = 1$

Solution

$xy^2 + y\sin(x) + y^3 = 1$

at $x = 0$

$y^3 = 1 \quad \rightarrow \quad y = 1$

$y^2 + x \cdot 2yy' + y' \sin(x) + y\cos(x) + 3y^2y' = 0$

$p(0,1)$

$1 + 0 + 0 + 1 + 3y' = 0$

$y' = -\frac{3}{2}$

$p(0,1) \qquad m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{2}{3}x$

Homework

	Find y' Find $\frac{dy}{dx}$
1	$\sin(x^2y^2) = x$
2	$\cos(x + 2y) = 0$
3	$x = \sin^2 y$
4	$x^2 = \frac{\cos y}{1 + \csc y}$
5	$f(x) = \sqrt{2x+1}\ \sec^3(5 - x^2)$ 41 July 19, 2007
6	$\sqrt{1 + \sin^3(xy^3)} = y$

1	9 December 12, 1996 If y is a function of x defined implicitly by $\cos(xy) + 2y + y^2 \tan x = 3$ then find the equation of the tangent line to the graph of y when $x = 0$
2	6 9 May 1996 Find an equation of tangent line to the graph of $y + y^2 \sin x + \sin(xy) = 1$ at $x = 0$
3	Find an equation of tangent lines to the graph of $x \sin y + \cos x + y^2 = 5$ when $x = 0$
4	4 15 December 1994 Find the equation of the normal line (lines) to the graph $xy + \tan x + y^2 = 9$ at the points on the graph where $x = 0$
5	11 May 15, 1997 & 13 May 15, 1997 Find an equation of the normal line to the graph of the following equation $x \sin y + xy + y - \pi = 0$ at the point whose x-coordinate is 0
6	Find the equation of the normal line to the graph $x^2y + \sin y = 2\pi$ at the point whose x-coordinate is 2π
7	23 April 27 , 2000 Find $\frac{dy}{dx}$ if y is implicitly defined by $x \tan\left(\frac{y}{x}\right) - 1 = 0$
8	Find an equation of the tangent line to the curve $3y^3 + 4xy - x^2 \sin y = 3$ at the point $P(0, 1)$

Homework

9	22 December 7, 1999 Find an equation of tangent line to the graph of $y + y^2 \tan x + \sin(xy) = 1$ at $x = 0$
10	33 May 6, 2004 Find an equation of tangent line to the graph of $\tan^2(x) - \cos(2x) - y^3 = 0$ at the point whose x-coordinate is π
11	31 July 31st , 2003 Find an equation of the normal line to the graph of $y = x^2 + x \sin y + \frac{\pi}{2}$ at $x = 0$
12	Find an equation of the normal line at $x = 0$ to the graph of $y^3 + y \sin(x) - \cos(xy^2) = 0$
13	28 Dec 20, 2001 Find an equation of the tangent line to the graph of $x^2y + \sin(xy + y^2) = x + 2$ at the point whose y-coordinate is 0
14	Find an equation for tangent line to the graph of the function given by the equation $x \cos y + y \cos x = \frac{\pi}{2}$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$
15	11 August 11, 1994 A Find the equation of the tangent line to the graph of the equation $\sqrt{xy} + \tan(1 - x^2) = 1$ at the point whose x −coordinate is 1
16	7 June 17, 1993 Let y be a function of x defined implicitly by $x \sin 2y = y \cos 2x$ Find the equation of the normal line to the graph of y at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
17	12 July 25, 1996, 1995 & 7 July 25, 1996 Find the equation of the tangent line to curve $1 + 16x^2y = \tan(x - 2y)$ at the point $\left(\frac{\pi}{4}, 0\right)$
18	1 December 3, 1992 Find an equation for tangent line to the graph of the function given by the equation $x \sin y + \cos x + y^2 = 5$ when $x = 0$

Homework

19 Find an equation of the tangent line to the graph of function $f(x) = \sec x \tan x + 1$ at $x = 0$

20 3 December 30, 1991
If $2y + \sin(xy) = -1 + \tan x$. Find the equation of the tangent line to the graph of y at $x = 0$

21 Find the equation of normal line to the graph of the function given by the equation $3y^4 + 4x - x^2 \sin x = 4$ at the point $(1, 0)$

22 10 June 6, 1994
Find an equation of the line normal to the graph of $\cos(x + y) + x^2y^2 - x = 0$ at the point $\left(0, \frac{\pi}{2}\right)$

23 37 May 4, 2006
Find an equation of the normal line at $x = 1$, to the graph of
$x^2y + \sin(xy - y) = 2$.

24 46 August 1, 2009
Find an equation of the normal line to the curve $\sec(x^2y) + \sqrt{x^2 + y} - x = 3$ at $x = 0$

25 Find the equation of the normal line to the graph
$y + \sqrt{1 + xy} + \tan(xy) = 1$ at the point whose x-coordinate is 0

26 34 July 22, 2004
Find an equation for the tangent line to the graph of $y^2 = x^3y^2 - x \sin y$ at the point $P(1, \pi)$

27 48 Sunday 9 May 2010
Find an equation of the tangent line to the curve
$x^2y^2 - 2x = 4 - 4y$
at the point $(2, -2)$. [4 marks]

28 49 July 24, 2010
(3 Points) Find an equation of the normal line to the graph of the equation
$(x^2 + y^2)^2 = 5xy + 15$
at the point $P(1, 2)$.

Homework

29	50 22 December 2010 (3 pts.) Suppose a curve is given by $x^2y + 5 = -\frac{1}{2}xy^3$. Find the slope of the tangent line to the curve at the point $(2, -1)$
30	51 8 May 2011 [3 pts.] Find an equation of the tangent line to the curve $x^3 + 2y^3 = 4xy + 2$. at the point $P(2, 1)$
31	31 July 31st , 2003 Find an equation of tangent line to the graph of $\sin^3(xy) + \pi = y + x$ at the point whose y-coordinate is 0
32	38 July 17, 2006 Find an equation of the normal line to the curve : $\tan(xy) + \sqrt{x+y} = 1$ at $x = 0$
33	Find $\frac{dy}{dx}$ if $\tan xy = xy$

31	31 July 31st , 2003 Find an equation of tangent line to the graph of $\sin^3(xy) + \pi = y + x$ at the point whose y-coordinate is 0

Solution

$\sin^3(xy) + \pi = y + x$

at $y = 0$

$x = \pi$

$p(\pi, 0)$

$3\sin^2(xy) \cdot \cos(xy) \cdot [y + xy'] = y' + 1$

$p(\pi, 0)$

$0 = y' + 1 \quad \Rightarrow \quad y' = -1$

$p(\pi, 0) \qquad m = -1$

$y - y_1 = m(x - x_1)$

$y - 0 = -(x - \pi)$

$y + x + \pi = 0$

32	38 July 17, 2006 Find an equation of the normal line to the curve : $\tan(xy)+\sqrt{x+y}=1$ at $x=0$

Solution

$\tan(xy)+\sqrt{x+y}=1$

at $x=0$

$0+\sqrt{y}=1 \quad \Rightarrow y=1$

$\sec^2(xy)\cdot[y+xy']+\dfrac{1+y'}{2\sqrt{x+y}}=0$

$p(0,1)$

$1\cdot[1+0]+\dfrac{1+y'}{2(1)}=0$

$2+1+y'=0 \quad \rightarrow \quad y'=-3$

$p(0,1) \quad ,m=\dfrac{1}{3}$

$y-y_1=m(x-x_1)$

$y-1=\dfrac{1}{3}x$

$3y-3=x$

$3y-x-3=0$

33	Find $\dfrac{dy}{dx}$ if $\tan xy=xy$

Solution

$\tan xy=xy$

$\sec^2(xy)\cdot[y+xy']=y+xy'$

$y\sec^2(xy)+xy'\sec^2(xy)=y+xy'$

$xy'\sec^2(xy)-xy'=y-y\sec^2(xy)$

$y'=\dfrac{y-y\sec^2(xy)}{x\sec^2(xy)-x}$

$=\dfrac{y(1-\sec^2(xy))}{x(\sec^2(xy)-1)}$

$=-\dfrac{y}{x}$